International Journal of Theoretical and Applied Science **3**(1): 43-44(2011) **ISSN : 0975-1718**

Fixed point theorem in psedo compact Tichonov space

Rajesh Shrivastava*, Animesh Gupta*, R.N. Yadava** and S.S Rajput***

**Department of Mathematics, Govt. Science and Commerce Benazir College Bhopal, (M.P.)*

***Ex-Director Gr. Scientist and Head, Resource Development Centre, AMPRI, Bhopal, (M.P.)*

****Department of Mathematics, Govt. P.G. College, Gadarwara, (M.P.) India*

(Received 28 Dec., 2010, Accepted 12 Jan., 2011)

ABSTRACT : In this paper, the concepts of compact metric space and psedo compact tichnov space has been introduced. We have proved some fixed point theorems for the self mapping satisfying a new contractive conditions in compact metric spaces and pseudo compact metric spaces.

Keywords : Fixed point, Compact Metric space, psedo compact Tichnov space, self mapping.

AMS Classification: 47H10.

I. INTRODUCTION

There are several generalizations of classical contraction mapping theorem in Bansch space [1]. In 196 Edelstein [2] established the existence of a unique fixed point of a self map T of a compact metric space satisfying the inequality $d(Tx,Ty) < d(x,y)$. Which is generalization of Banach. In the past few years a number of authors such as Iseki [3], fisher [4] Bhardwaj [5] have proved the number of interesting result on compact metric space. We are finding some fixed point theorems in psedo compact tichonov spaces.

Recently, Park [8] introduced the notion of intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces. Kutukcu *et. al*. [2] introduced the notion of intuitionistic Menger spaces with the help of t-norms and t-conorms as a generalization of Menger space due to Menger [3]. Recently in 2009, using the concept of subcompatible maps, Bouhadjera et. al. [1] proved common fixed point theorems in metric space. Using the concept of weakly compatible maps in intuitionistic Menger space, Pant et. al. [7] proved a common fixed point theorem for six self maps without appeal to continuity.

II. PRELIMINARIES

Definition A. Let T be a self continuous mapping. A space X is called a fixed point space, if every continuous mapping T of X into itself, has a fixed point.

Definition B. Pseudo-compact tichonov space : A topological space X is said to be Pseudo-compact space, if every real valued continuous function on X is bounded. It may be noted that every compact space is psedo compact, but converges may not be true. Tichonov space , we mean a completely regular Housdroff space.

Now we prove, following theorems.

Theorem 1. Let P be a Psedo compact Tichonov space and d be a non negative real valued continuous function such that $d : P \times P \to R^+$, satisfying the condition,

(*i*) $d(x, x) = 0 \ \forall \ x \in X$

(*ii*) $d(x, z) \leq d(x, y) + d(y, z) \ \forall \ x, y, z \in X$

(*iii*) $d(Tx, Ty) \leq \alpha\{d(x, Tx)\} + d(y, Ty) + \beta\{d(x, Ty) + d(y, Tx) + \gamma\, d(x, y)\}$

where $\alpha, \beta, \gamma \geq 0$ *such that* $0 \leq \alpha + \beta + \gamma < 1$ *and*

$$0 \leq \frac{\alpha+\beta+\gamma}{1-\alpha-\beta} < 1$$

Then T has unique fixed point in P.

Proof. We define a function $\varphi : P \to R^+$ by $\varphi(P) = d(p, Tp)$, for all $p \in P$, where R^+ is the set of positive real numbers. It is clear that φ is continuous generated by the composition of two continuous function *T and d.* Since P is psedocompact Tichonove space. Every real valued continuous function over P is bounded and attend its bounds.

Thus there exists a point $u \in P$ such that $\varphi(u) = \inf[\varphi(p) : p \in P]$. Now we suppose that u is a fixed point for T, if not;

Let us $\varphi(Tu) = d(Tu, T^2u)$

From above

$$d(Tu, T^2u) \leq \alpha\{d(u, Tu) + d(Tu, T^2u)\} + \beta\{d(u, T^2u) + d(Tu, Tu)\} + \gamma\, d(u, Tu)$$

$$d(Tu, T^2u) < \alpha\{d(u, Tu) + d(Tu, T^2u)\} + \beta\{d(u, Tu) + d(Tu, T^2u)\} + \gamma\, d(u, Tu)$$

$$(1-\alpha-\beta)\, d(Tu, T^2u) \leq (\alpha+\beta+\gamma)\, d(u, Tu)$$

$$d(Tu, T^2u) < \frac{\alpha+\beta+\gamma}{(1-\alpha-\beta)}\, d(u, Tu)$$

$$\varphi(Tu) \leq \varphi(u)$$

u is a fixed point of T in P.

Uniqueness. Let us assume that w is another fixed point different from u in P, so that

$d(u, w) = d(Tu, Tw)$.

From (3),

$d(Tu, Tw) \leq \alpha \{d(u, Tu) + d(w, Tw)\} + \beta \{d(u, Tw) + d(w, Tu)\} + \gamma d(u, w)$

$d(Tu, Tw) \leq (2\beta + \gamma)\, d(u, w)$

which contradiction;

u is unique fixed point of T

Theorem 2. Let T be a continuous mapping of a compact metric space X into itself, satisfying the condition;

$$d(Tx, Ty) \leq \alpha \frac{d(x,y)\,[d(x,Tx) + d(y,Ty)]}{d(x,Ty) + d(y,Tx)}$$

For all $x, y \in X$, $x \neq y$ and $0 \leq \alpha < 1$. then T has unique fixed point.

Proof. We define a function $\varphi : X \to R^+$ by $\varphi(x) = d(x, Tx)$, for all $p \in X$, where R^+ is the set of positive real numbe. It is clear that φ is continuous generated by the composition of two continuous function *T and d.* Since X is compact space. Every real valued continuous function over X is bounded and attend its bounds.

Thus there exists a point $u \in X$ such that $\varphi(x) = \inf[\varphi(x) : x \in X]$. Now we suppose that u is a fixed point for T, if not;

Let us $\varphi(Tu) = d(Tu, T^2u)$

From (4),

$$d(Tu, T^2u) \leq \alpha \frac{d(u,Tu)\,[d(u,Tu) + d(Tu,Tu)]}{d(u,T^2u) + d(Tu,Tu)}$$

$$d(Tu, T^2u) \leq \alpha \frac{d(u,Tu)\,[d(u,Tu) + d(Tu,T^2u)]}{d(u,T^2u) + d(Tu,Tu)}$$

$d(Tu, T^2u) \leq \alpha\, d(u, Tu)$

which contradiction;

u is fixed point of T in X

Uniqueness. Let us assume that w is another fixed point different from u in P, so that

$d(u, w) = d(Tu, Tw)$

$$d(Tu, Tw) \leq \alpha \frac{d(u,w)\,[d(u,Tu) + d(w,Tw)]}{d(u,Tw) + d(w,Tu)}$$

$d(Tu, Tw) \leq 0$

Which contradiction;

u is unique fixed point *of T* in *ZX*.

REFERENCES

[1] Banach: Surles operation dansles ensembles abstraites etleur application integrals. *Fund. Math.*, **3:** 133-181 (1922).

[2] Edekstgein M: an extension of Banach’s Constraction principal, *Proc. Amer, math, soc.*, **12:** 7-10 (1961).

[3] Iseki K: simple application of Reich’s fixed point theorem. *Math. Notes kobe university,* **5:** (1997).

[4] B.Fisher: on three fixed point mappings for compact metric space, *IJPAM* **8**(4)**:** 479-481(1977).

[5] R.Bhardwaj, some fixed point theorems in compact space and Pseudo-compact Tichonov space, *IJMA*, **2:** 543-550 (2008).

[6] Pathak J.K, some theorems on fixed points in Pseudo compact Tichonov space. *Indian J. Pure and Applied Math.j* **15**(5): 180-186 (1986).